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Swamp Optics Tutorial

Interferometric Autocorrelation

A pulse-measurement method that combines quantities related to the autocorrelation and spectrum in a single data trace is the *interferometric autocorrelation*, often called *phase-sensitive autocorrelation* and the *fringe-resolved autocorrelation (FRAC)*. It was introduced by Jean-Claude Diels in 1983, and it became popular very quickly. It involves measuring the second-harmonic energy vs. delay from an SHG crystal placed at the output of a Michelson interferometer (see below). In other words, it involves performing an autocorrelation measurement using collinear beams, so that the second harmonic light created by the interaction of the two different beams combines coherently with that created by each individual beam. As a result, interference occurs due to the coherent addition of the several beams, and interference fringes occur vs. delay. This is in contrast to the usual autocorrelation, which is often referred to as the *background-free autocorrelation* or *intensity autocorrelation* (see the Swamp Optics tutorial on this subject for more information) when FRAC is also being discussed.

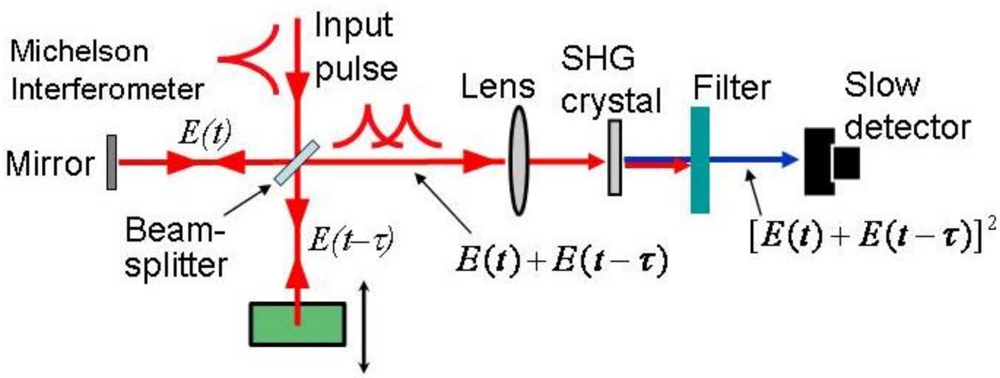

Fig. 1. Experimental layout for the fringe-resolved autocorrelation (FRAC).

The expression for the FRAC trace is:

$$I_{FRAC}(\tau) = \int_{-\infty}^{\infty} \left| \left[E(t) + E(t-\tau) \right]^2 \right|^2 dt$$

$$= \int_{-\infty}^{\infty} \left| E(t)^2 + 2\,E(t)\,E(t-\tau) + E(t-\tau)^2 \right|^2 dt$$

Note that, if the $E(t)^2$ and $E(t\text{-}\tau)^2$ terms were removed from the above expression, we'd have only the cross term, $2E(t)E(t\text{-}\tau)$, which yields the usual expression for background-free intensity autocorrelation (see the tutorial on Intensity Autocorrelation). These new terms, integrals of $E(t)^2$ and $E(t\text{-}\tau)^2$, are due to SHG of each individual pulse. And their interference, both with each other and with the cross term, will yield the additional information in the FRAC that is not present in the usual autocorrelation. Indeed, the interference of these new terms with each other will yield an interferogram of the second harmonic of the pulse.

Expanding the above expression:

$$I_{FRAC}(\tau) = \int_{-\infty}^{\infty} \left\{ I(t)^2 + I(t-\tau)^2 \right\} dt \quad + \quad \int_{-\infty}^{\infty} \left\{ I(t) + I(t-\tau) \right\} \mathrm{Re}\left\{ E(t)\, E^*(t-\tau) \right\} dt \quad +$$

$$\int_{-\infty}^{\infty} \mathrm{Re}\left\{ E(t)^2\, E^*(t-\tau)^2 \right\} dt \quad + \quad \int_{-\infty}^{\infty} I(t)\, I(t-\tau)\, dt$$

In words,

$I_{FRAC}(\tau)$ = Constant + Modified interferogram of $E(t)$ +

Interferogram of the 2nd harmonic of $E(t)$ + Autocorrelation of $I(t)$

Thus, the FRAC contains a constant, the autocorrelation, something akin to the interferogram (which we refer to here as the "modified interferogram" due to the additional factor, $I(t) + I(t-\tau)$, not present in the interferogram), and the interferogram of the pulse second harmonic. Examples of the FRAC are shown below.

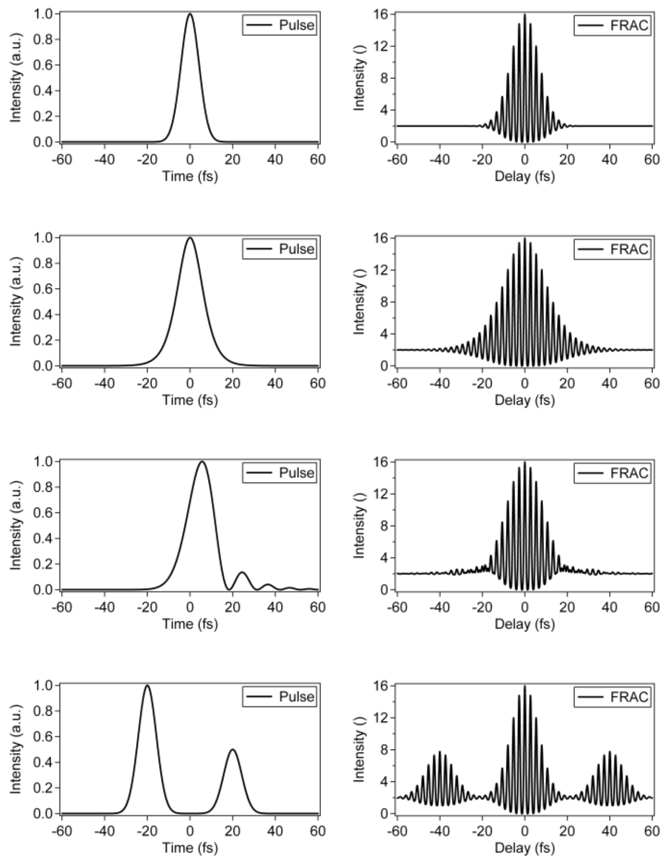

Fig. 2. Pulses and their FRAC traces. Top row: A 10-fs Gaussian intensity. Second row: A 7-fs sech^2 intensity. Third row: A pulse whose intensity results from 3^{rd}-order spectral phase. Fourth row: A double pulse. Note that the satellite pulses due to third-order spectral phase, which were invisible in the intensity autocorrelation, actually can be seen in the wings of the FRAC trace.

We can now dissect this quantity and try to understand it.

Let's start with the constant term, which isn't very interesting, but it is useful for verifying the validity of a measurement. The peak-to-background ratio in a FRAC trace is 8. If it isn't, then there's a problem with the measurement, and it's necessary to redo it. In any case, this information won't help us determine the pulse.

Now consider the last term. It's just the intensity autocorrelation. This is helpful, but not that helpful.

Now consider the two interferogram terms. Recall that interferograms yield fringes with respect to delay with the frequency of the light involved. And, in the FRAC trace, there are two interferograms, with such fringes. The fringes in the modified interferogram of E(t) occur at frequency ω. And the fringes in the interferogram of the 2nd harmonic of E(t) occur at frequency 2ω. As a result, except for extremely short pulses of only a few cycles, the various terms can be distinguished by their different carrier frequencies.

Recall that the interferogram is the inverse Fourier transform of the spectrum. Thus, the interferogram of the 2nd harmonic of $E(t)$ simply yields the spectrum of the 2nd harmonic of $E(t)$.

Now consider the modified interferogram of $E(t)$. This term doesn't correspond to any well-known or intuitive quantity. In the limit that the distortions are mostly in the phase, however, the quantity, $I(t) + I(t-\tau)$, is slowly varying compared to $\mathrm{Re}\{E(t)E^*(t-\tau)\}$, so the remaining integral reduces to the simple interferogram of $E(t)$. In this limit, then, this term is simply equivalent to the pulse spectrum.

So when the distortions are mostly in the phase:

$$I_{FRAC}(\tau) \approx \text{Constant} + \text{Interferogram of } E(t) + \text{Interferogram of } E^2(t) + \text{Autocorrelation of } I(t)$$

So what does all this interesting information do for us? Does the FRAC completely determine the pulse field? Unfortunately, no study has been made of what can be retrieved from the FRAC and what ambiguities arc present (besides the obvious direction-of-time ambiguity).

Nagunuma has shown that, if the pulse spectrum or interferogram is also included, there is in principle sufficient information present to fully determine the pulse field (except for the direction of time). He also presented an iterative algorithm to find the field. No study has been published on this algorithm's performance, however, and it is rarely used. Researchers who have tried it have found that it tends to stagnate.

Chung and Weiner have shed some light on the issue of how well FRAC determines pulses by calculating FRAC traces for the pairs of pulses. And they found that the resulting traces of the pairs of pulses had very similar, although not identical, FRAC traces. See Fig. 3 below.

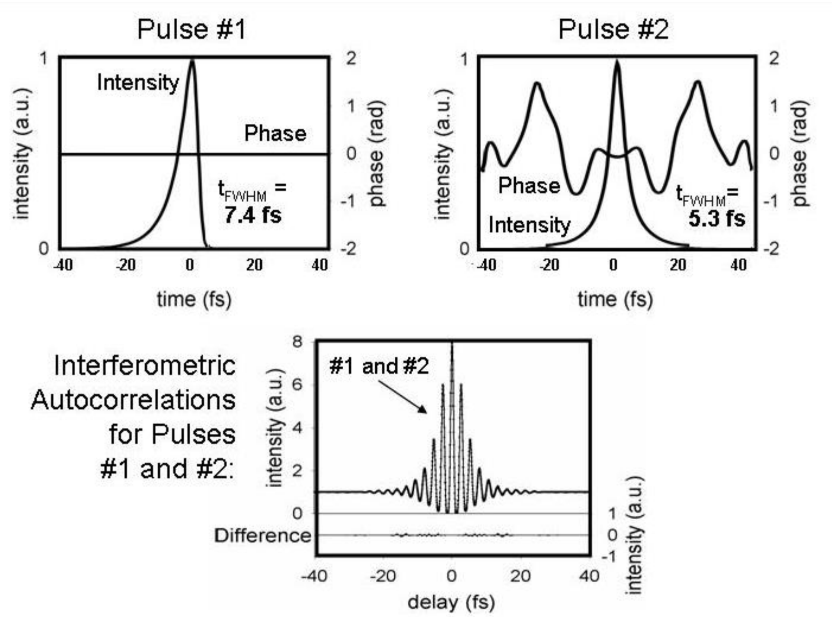

Fig. 3. Two pulses (top) and their FRAC traces (bottom). The difference between the two FRAC traces is also plotted below. Note that, despite the significant differences between the two pulses, their FRAC traces are nearly identical. This pair of pulses with nearly identical FRAC traces is only one of infinitely many such pairs, most of which have never been tabulated.

On the other hand, Diels and coworkers showed that additional information could be gleaned by including a second FRAC measurement—actually a fringe-resolved *cross*-correlation—in which some glass is placed in one of the interferometer arms. This breaks the symmetry and yields an asymmetrical trace. Then, assuming that the dispersion of the glass is known, Diels and coworkers showed that the two FRAC traces could be used to completely determine the pulse field in a few cases. Again, however, no study has been published on this algorithm's performance. On the other hand, Diels gave this method a memorable name, *The Femto-Nitpicker*.

You can read more about FRAC and most other pulse-measurement techniques in Rick Trebino's book, *Frequency-Resolved Optical Gating: The Measurement of Ultrashort Laser Pulses*, Kluwer Academic Publishers, 2002.

Today, more than twenty-five years after its introduction, interferometric autocorrelation, like its cousin, intensity autocorrelation, is considered obsolete. Better, more powerful techniques exist that are easier to perform. See the FROG tutorial.

About Swamp Optics

Founded in 2001, Swamp Optics, LLC, offers cost-effective quality devices to measure ultrashort laser pulses. It specializes in frequency-resolved optical gating (FROG) and GRENOUILLE (an experimentally simple version of FROG), the gold standards for measuring the time-dependent (or, equivalently, frequency-dependent) intensity and phase of an ultrashort pulse.

Swamp Optics also sells an innovative pulse compressor.

For more information, visit us on the Web at www.swampoptics.com.