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Swamp Optics Tutorial

SPIDER

What is SPIDER and how does it work?

In the early 1990s, two groups independently introduced—and "demonstrated"—new techniques based on "spectral shearing interferometry (SSI)" for measuring pulses. [V. Wong and I.A. Walmsley, *Pulse-shape measurement using linear interferometers*, in *Proceedings of the SPIE*, 1994, p. 254-67; Lai, M. and J.C. Diels, *Complete Diagnostic of Ultrashort Pulses Without Nonlinear Process,* Opt. Commun, 1992. **88**(4,5,6): p. 319-325]. SSI involved spectrally dispersing two replicas of the pulse, displacing one relative to the other in space, and measuring the resulting interference fringes produced by the interference of different wavelengths vs. position. A big advantage of SSI was that it had a simple direct inversion algorithm. Also, it did away with the nonlinear-optical medium, which limited the sensitivity of autocorrelation and FROG. It was also very easy to set up and align.

SSI had only one drawback: it didn't actually work. It required interfering beams of different colors at a given point in space, and only beams of the same color interfere. Walmsley later published a paper proving that such methods couldn't ever work [V. Wong and I.A. Walmsley, *Analysis of Ultrashort Pulse-Shape Measurement Using Linear Interferometers,* Opt. Lett., 1994, **19**(4): p. 287-289.].

But Walmsley didn't give up on the idea of spectral-shearing interferometry. In 1999, he introduced a new SSI technique, which he called SPectral Interferometry for Direct E-field Reconstruction, or SPIDER and based on the combination of SSI and a technique called spectral interferometry (SI). SI involves measuring the spectrum of the sum of two pulses (separated by a few pulse lengths in time), which has spectral fringes from which one can extract the spectral-phase difference between the two pulses. In other words, if you know one of the pulses, it's then possible to determine the other. The SI spectrum is given by:

$$S_{SI}(\omega) = S_{ref}(\omega) + S_{unk}(\omega) + 2\sqrt{S_{ref}(\omega)}\sqrt{S_{unk}(\omega)}\cos[\varphi_{ref}(\omega) - \varphi_{unk}(\omega) + \omega\tau]$$

where τ is the pulse separation in time, and we have used subscripts "unk" and "ref" to indicate the unknown and reference pulses. Unfortunately, performing SI using the pulse and itself yields no useful information because the spectral phase simply cancels out. Thus, SI is only useful if a reference pulse has already been characterized. And that's not helpful in most cases.

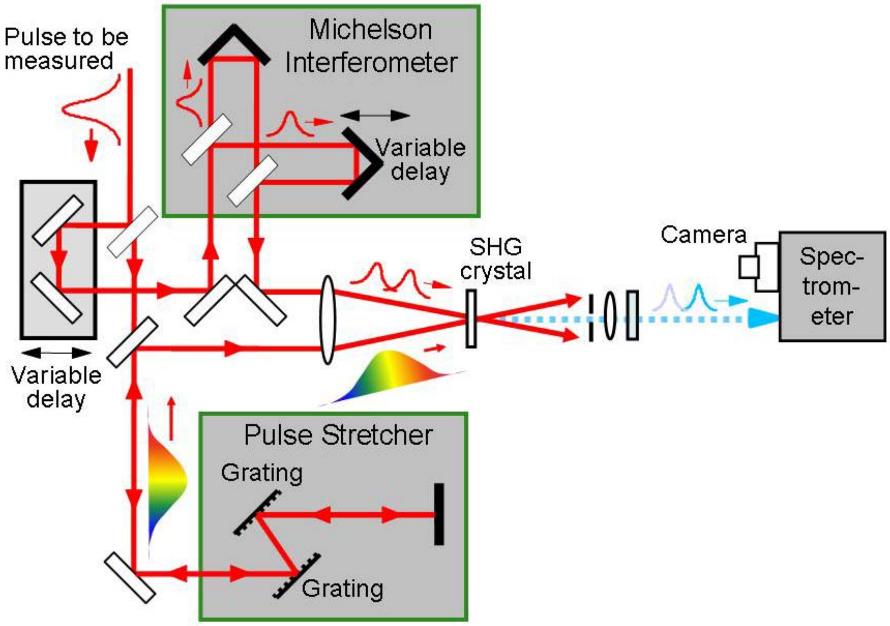
SPIDER, on the other hand, involves spectrally shifting one replica of a pulse by $\delta\omega$ with respect to another. Thus, the reference pulse phase is $\varphi(\omega+\delta\omega)$. This yields what we might call the SPIDER phase (the argument of the cos):

$$\phi_{SPIDER}(\omega) \;=\; \varphi(\omega+\delta\omega) - \varphi(\omega) - \omega\tau \;=\; \delta\omega\frac{d\varphi}{d\omega} - \omega\tau \;=\; \delta\omega\,\tau_{gr} - \omega\tau$$

which can be simplified by noting the similarity of the first two terms to the numerator of a derivative, and that derivative is the group delay, τ_{gr}, that is, the arrival time as a function of frequency. It is this quantity that is found in SPIDER.

As in the earlier version of SSI, a direct inversion algorithm determined the pulse from spectral fringes. Unlike it, however, it can easily be shown that SPIDER should work if all the parameters in the measurement are known. In fact, the pulse chirp could be obtained directly from the SPIDER spectral-fringe spacing.

SPIDER involves a complex apparatus that requires splitting the pulse to be measured into two replicas with a well-defined separation in time. A nonlinear-optical process then color-shifts each of these pulses by a different amount (the spectral shear), and a spectrometer measures the spectrum of the sum of the two pulses. The apparatus is shown below.



SPIDER's main advantage is that it yields the unknown pulse phase using a direct algorithm and so is fairly fast. With the advent of fast computers, however, this advantage over iterative approaches (such as FROG and GRENOUILLE) is no longer significant. Some groups still use SPIDER, so it's a good idea to look at it more closely, and there are some issues that must be understood before undertaking SPIDER measurements. *Indeed, it has recently been shown that SPIDER has some serious accuracy problems, and, as a result, it should only be used by those who have tremendous expertise in its use. In the past year or so, many new versions of SPIDER have been introduced to solve these problems, but so far none has proven successful.*

How accurate is SPIDER?

It was recently shown that SPIDER has serious calibration/accuracy issues. Because SPIDER measures the ***derivative*** of the pulse phase, the SPIDER linear phase yields the pulse ***quadratic*** spectral phase (linear chirp). *Unfortunately, the separation, τ, of the double pulse required in SPIDER has precisely the same effect.* ***So any error in the internally generated double-pulse separation will yield a large error in the measured pulse chirp.***

To see this, note that the SPIDER spectral-fringe phase is:

$$\phi_{SPIDER}(\omega) \;=\; \varphi(\omega+\delta\omega)-\varphi(\omega)-\omega\tau \;=\; \delta\omega\,\frac{d\varphi}{d\omega}-\omega\tau \;=\; \delta\omega\,\tau_{gr}-\omega\tau$$

An error in τ of $\delta\tau$ will correspond to an error in the group delay, $\delta\tau_{gr}$:

$$\delta\omega\,\delta\tau_{gr} = \omega\,\delta\tau$$

or:

$$\delta\tau_{gr} = \frac{\omega}{\delta\omega}\,\delta\tau$$

The group delay errors at the maximum and minimum frequencies in the pulse spectrum are then:

$$\left(\delta\tau_{gr}\right)_{\min} = \frac{\omega_{\min}}{\delta\omega}\,\delta\tau \qquad\qquad \left(\delta\tau_{gr}\right)_{\max} = \frac{\omega_{\max}}{\delta\omega}\,\delta\tau$$

and the error in the pulse length will be:

$$\delta\tau_p = \frac{\omega_{\max}-\omega_{\min}}{\delta\omega}\,\delta\tau \qquad \text{or} \qquad \delta\tau_p = \frac{\Delta\omega}{\delta\omega}\,\delta\tau$$

This expression can be rewritten in terms of ratios:

$$\frac{\delta\tau_p}{\tau_p} \;=\; \frac{\Delta\omega}{\delta\omega}\,\frac{\tau}{\tau_p}\,\frac{\delta\tau}{\tau} \;\approx\; 100\times 100\times\frac{\delta\tau}{\tau}$$

using typical numbers for the first two ratios from publications using SPIDER.

Unfortunately, the relative accuracy of the separation, $\delta\tau/\tau$, is almost always > 10^{-4}. So the error in most SPIDER-measured pulse lengths is 100%. In order to make an accurate SPIDER measurement, the separation accuracy must be ~ 10^{-6}! This is often just a few attoseconds and is always quite difficult. Devices capable of measuring pulse separations with this accuracy are more complex than the SPIDER itself!

Some SPIDER measurements have been performed using 10, rather than 100, for the first two ratios, but the calibration is still difficult! Also, in this case, only ten points exist in the trace and resulting measured spectral phase, and it isn't possible to know what the phase is in between them (this is not the case in FROG). This opens up the possibility that more complex phases could also yield the same trace.

Currently, many new versions of SPIDER are appearing in the literature in an attempt to fix this problem and other issues in SPIDER. Perhaps one will work well. But so far none has succeeded. Some claim that their new SPIDER method solves this problem, but no one has actually shown that it does in fact do so. Indeed, those that we have considered do not appear to solve the problem.

What do SPIDER traces look like?

We can obtain an idea as to why SPIDER has these problems by simply looking at SPIDER traces for various pulses. Unfortunately, it seems that no one has ever published a catalog of SPIDER traces for common pulses and distortions—something that developers of most other techniques do immediately. How do SPIDER traces differ for different distortions?

Let's plot traces for a flat-phase pulse and another with the same spectrum, but which has broadened in time by a factor of two due to simple linear chirp. How different are the SPIDER traces? They should be very different because a factor of two difference in pulse length is a lot. Even autocorrelation traces easily distinguish a pulse from another that's twice as long.

The ideal SPIDER trace is given by:

$$S_{SI}(\omega) = S(\omega) + S(\omega+\delta\omega) + 2\sqrt{S(\omega)}\sqrt{S(\omega+\delta\omega)}\cos[\varphi(\omega+\delta\omega) - \varphi(\omega) + \omega\tau]$$

where $S(\omega)$ is the pulse spectrum, $\varphi(\omega)$ is the pulse spectral phase, and $\delta\omega$ is the frequency shift generated in the SPIDER device. The SPIDER phase is the argument of the cos:

$$\phi_{SPIDER} = \varphi(\omega+\delta\omega) - \varphi(\omega) + \omega\tau = \delta\omega\frac{d\varphi}{d\omega} + \omega\tau$$

The $\omega\tau$ term is just due to the separation (τ) between the double pulse generated within the SPIDER and is independent of the pulse to be measured. The important term is the other one: $\delta\omega \, d\varphi/d\omega = \delta\omega \, \tau_{gr}$, where $\tau_{gr} = d\varphi/d\omega$ is the group delay. $\delta\omega \, \tau_g$ is the extra phase due to the pulse phase distortions, which is the purpose of a SPIDER measurement.

A simple look at this expression indicates that SPIDER could have trouble measuring pulses accurately because the second (uninteresting) term, $\omega\tau$, is *much larger* than the first term, $\delta\omega \, d\varphi/d\omega$, which contains the desired pulse information, $d\varphi/d\omega$. Let's write them in terms of the pulse length, τ_p, and the pulse bandwidth, ω_p. The pulse frequency, ω, is always greater than the bandwidth and usually > 10 ω_p. The pulse separation is usually ~ 100 τ_p. So the second term is approximately (10 ω_p) (100 τ_p) = 1000 $\omega_p \tau_p$. We'll consider near-transform-limited pulses ($\tau_{gr} < \tau_p$), which is the case for essentially all SPIDER measurements ever made. Finally, the frequency shift must always be << ω_p and is usually 0.01 ω_p, so the first term is less than (0.01 ω_p) (1 τ_p) = 0.01 $\omega_p \tau_p$:

$$\phi_{spider} = \underset{\underset{\sim\omega_p/100}{\nearrow}}{\delta\omega} \cdot \underset{\underset{<\tau_p}{\nearrow}}{\tau_{gr}} + \underset{\underset{>10\omega_p}{\uparrow}}{\omega}\,\underset{\underset{100\tau_p}{\nwarrow}}{\tau}$$

Thus the term we care about is 10^5 smaller than the uninteresting term (and over a SPIDER trace, its variation is still almost 10^5 smaller than the variation in the uninteresting term, which necessarily varies by several times ω_p). It seems difficult to imagine that one could measure this small term in the presence of the big one. Interferometer researchers would usually say that we'd need 10^5 fringes to see a phase term that's this small, and a typical SPIDER trace has 1000 times fewer fringes.

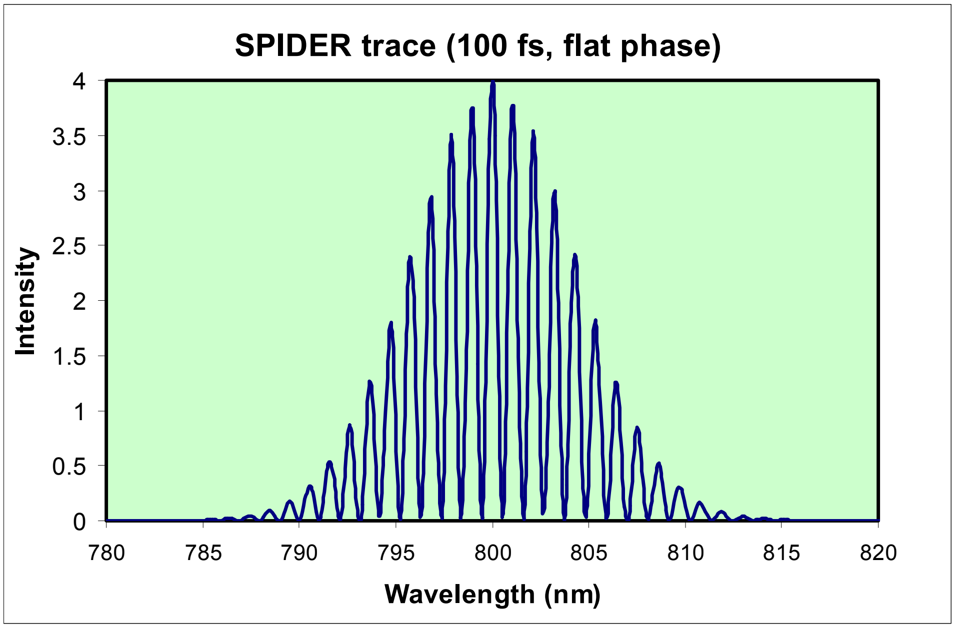
Worse, as mentioned earlier, in the presence of only linear chirp, the most important type of phase distortion, $\varphi(\omega) = \varphi_2 \times (\omega - \omega_0)^2/2$, the first term has precisely *the same form* as the larger uninteresting term (i.e., both are linear in ω):

$$\phi_{SPIDER} = \delta\omega\,\varphi_2 \times (\omega - \omega_0) + \omega T$$

and so would be even harder to distinguish from the larger term.

SPIDER for a pulse without chirp

Let's compute some SPIDER traces for very different pulses that we'd like to potentially measure and see how different they are. Computing a SPIDER trace for a flat-phase 100-fs pulse (i.e., a pulse with no chirp), we find:



We've used a 100-fs pulse with a bandwidth of 10 nm, a frequency shift of 0.1 nm, and a pulse separation of 2 ps, fairly typical numbers for SPIDER measurements.

SPIDER for a pulse with chirp

How much chirp is required to double the pulse length? And what does the SPIDER trace look like for it?

First, how much pulse phase, φ_2, is required to double a pulse length? From the Appendix, the extra phase required to double a Gaussian pulse due to chirp will be:

$$\varphi_2 = 2\sqrt{3} / \Delta\omega^2 = 2\sqrt{3}\,\lambda_0^4 / \left(2\pi c\,\Delta\lambda\right)^2 = \sqrt{3}\,\lambda_0^4 / 2\pi^2 c^2 \Delta\lambda^2$$

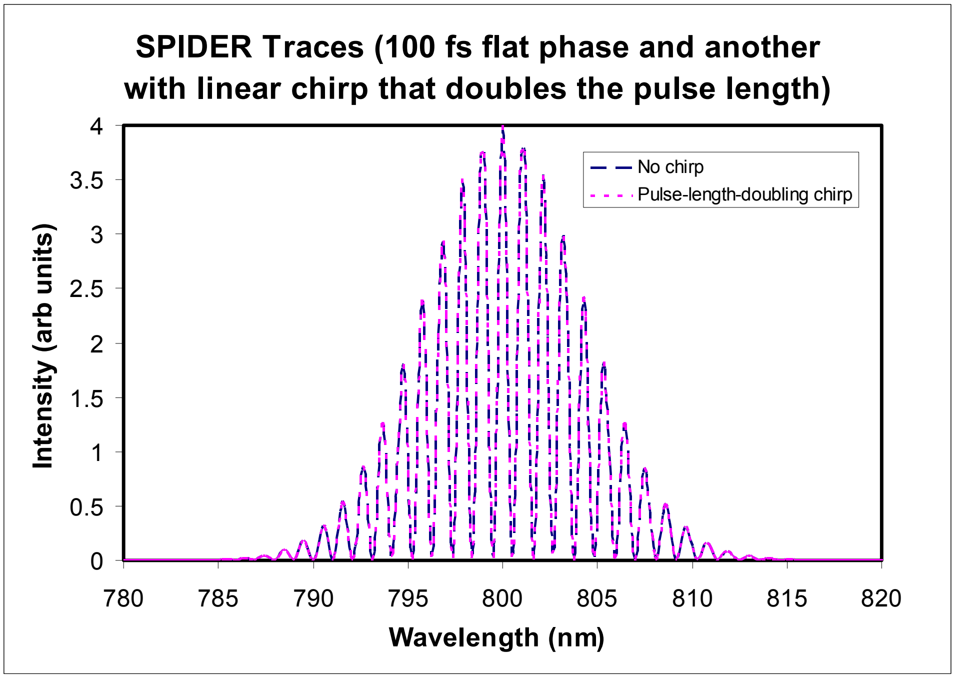
In this case, the extra SPIDER phase that we need to measure will be:

$$\begin{aligned} &= \delta\omega\,\varphi_2 \times (\omega - \omega_0) \\ &= \left[2\pi c\left(\frac{\delta\lambda}{\lambda_0^2}\right)\right][\varphi_2]\left[\frac{2\pi c}{\lambda} - \frac{2\pi c}{\lambda_0}\right] \end{aligned}$$

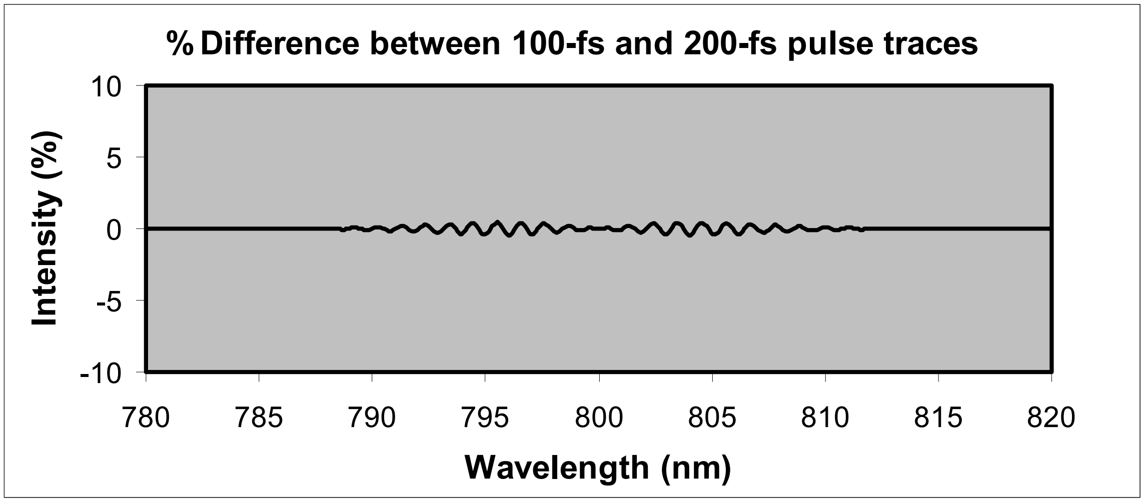
$$= \left[2\pi c\left(\frac{\delta\lambda}{\lambda_0^2}\right)\right][\sqrt{3}\,\lambda_0^4 / 2\pi^2 c^2 \Delta\lambda^2]\left[\frac{2\pi c}{\lambda}-\frac{2\pi c}{\lambda_0}\right]$$

$$=[2\sqrt{3}]\left[\delta\lambda\,\lambda_0 / \Delta\lambda^2\right]\left[\frac{\lambda_0}{\lambda}-1\right]$$

Now let's plot the SPIDER traces of both the flat-phase pulse (100 fs long) and this new one with the same spectrum, but with enough linear chirp to double the pulse length to 200 fs; we find:



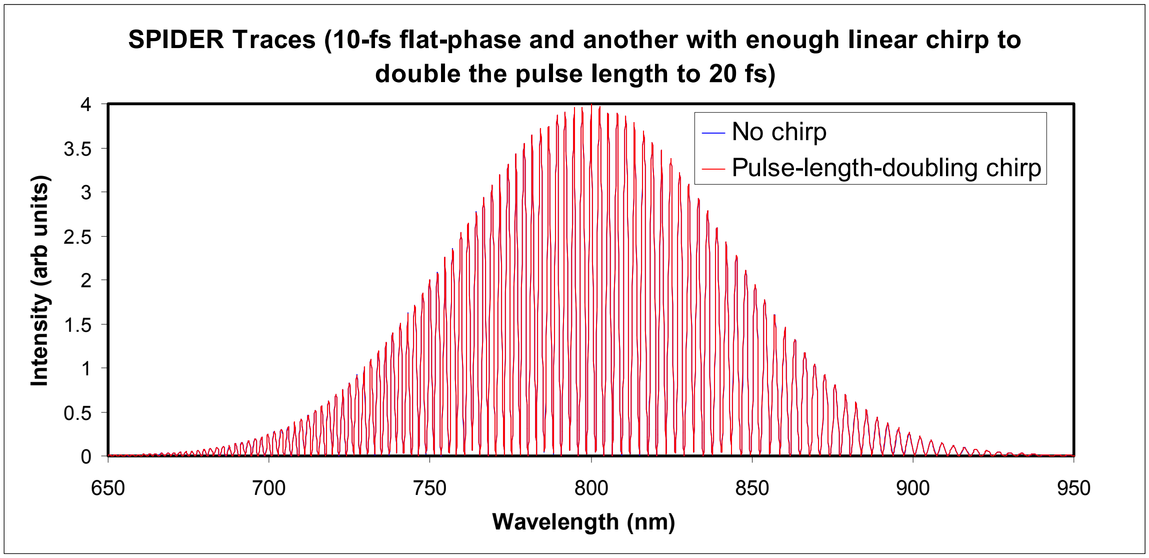
It's difficult to see the difference between the two traces, despite the fact that the two pulses are very different. The difference between these two traces is:



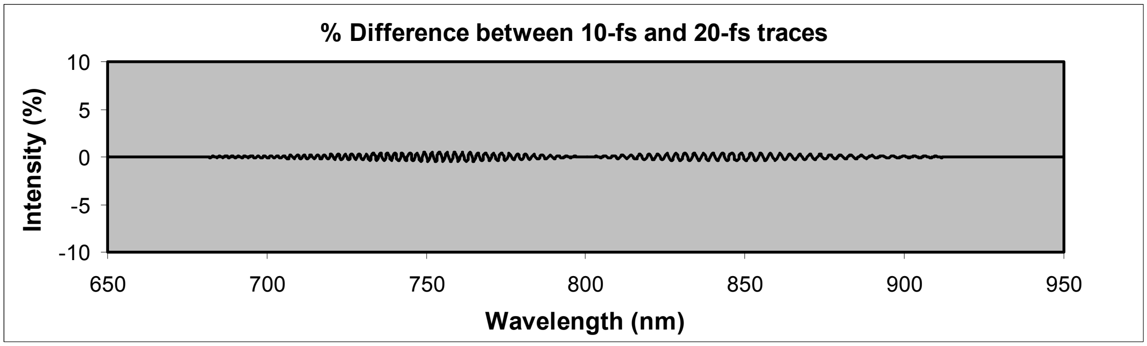
Thus, there is less than 1% difference between the two traces for these two very different pulses (which are 100 and 200 fs long, respectively).

This makes sense. Because adding linear chirp has the same effect in SPIDER as changing the double-pulse separation, and the phase contribution from the double-pulse separation is ~100,000 times larger, it's difficult to imagine that SPIDER can measure linear chirp very sensitively, especially since they have the same form (they're both linear in ω).

It's similarly difficult to distinguish between a 10-fs flat-phase Gaussian pulse and a 20-fs linearly chirped pulse with the same spectrum (for this simulation, we've used a 100-nm bandwidth, a 1-ps separation, and a 1-nm frequency shift, again typical numbers:



Again, they're indistinguishable to the eye, so here's the difference:



Again, there's less than 1% difference between the two traces.

Discussion and Conclusions

These ideal traces assume perfect measurements. In reality, the typical SPIDER trace only has about a 30% fringe visibility (which would reduce difference between the traces by a factor of ~ 3), and there's dark current and pixel-to-pixel variations in the camera responsivity, as well as other sources of noise, which would effectively mask such small differences. So it seems difficult to believe that SPIDER could be able to distinguish between one pulse and another that's twice as long. But we'd like SPIDER to be able to measure much smaller differences between pulses.

On the other hand, when a pulse doubles in length, a simple autocorrelation yields a trace that's twice as wide and very easy to distinguish from the shorter one.

Finally, as mentioned earlier, there has been some question as to how well SPIDER can operate in view of calibration difficulties, which have not been solved yet. But in this latter analysis, we've assumed that all calibration parameters are known *exactly*. So the issue discussed here is different, and any calibration problems would only seem to compound the problem.

In addition to these issues, SPIDER has some advantages, but many additional disadvantages.

Advantages:

Pulse retrieval is direct (i.e., non-iterative) and hence fast.
Minimal data are required: only one spectrum yields the spectral phase.
It naturally operates single-shot (although most other techniques also operate single shot).

Disadvantages:

Its apparatus is **very** complicated. It has 12 sensitive alignment parameters (5 for the Michelson; 4 in pulse stretching; 1 for pulse timing; 2 for spatial overlap in the SHG crystal; not counting the spectrometer).
Like spectral interferometry, it requires very high mechanical stability, or the fringes wash out.
Poor beam quality can also wash out the fringes, preventing the measurement.
It has no independent checks or feedback that the measurement is correct. Its minimal data prevent it from having any independent check on the measurement.
It cannot measure long or complex pulses: The maximum time-bandwidth product ever measured using SPIDER is ~ 3. (Spectral resolution is ~ 5 times worse than that of the spectrometer due to the need for spectral fringes.)
It has poor sensitivity due to the need to split and stretch the pulse *before* the nonlinear medium (unlike FROG and autocorrelation).
The pulse delay must be chosen for the particular pulse. And pulse structure can confuse it, yielding ambiguities.

You can read more about most other pulse-measurement techniques in Rick Trebino's book, *Frequency-Resolved Optical Gating: The Measurement of Ultrashort Laser Pulses*, Kluwer Academic Publishers, 2002.

While some groups that are expert in performing SPIDER measurements probably can maintain the high accuracy necessary to perform accurate SPIDER measurements, one should not undertake SPIDER measurements unless one is willing to become such an expert and probably write papers on a new version of SPIDER that one invents to solve the above problems. FROG and GRENOUILLE are much easier and more reliable. They also offer the much-needed feedback that the measurement was correctly made. See the FROG tutorial.

About Swamp Optics

Founded in 2001, Swamp Optics, LLC. offers cost-effective quality devices to measure and manipulate ultrashort laser pulses. It specializes in frequency-resolved optical gating (FROG), a method for measuring the time-dependent (or, equivalently, frequency-dependent) intensity and phase of an ultrashort pulse. FROG is rigorous, general, and relatively simple to implement; it has become a very successful technique, with many accomplishments.

Swamp Optics also sells inexpensive, alignment-free, and elegant pulse compressors.

For more information, visit us on the Web at Swamp Optics--Ultrashort Laser Pulse Measurement.

Appendix

How much linear chirp is required to double a pulse length?

Suppose the pulse field in the frequency domain is:

$$\tilde{E}(\omega) = \exp\left\{-\left[(\omega-\omega_0)^2/\Delta\omega^2\right]\right\}\exp\left[-\tfrac{1}{2}i\varphi_2(\omega-\omega_0)^2\right]$$

Its Fourier transform is:

$$E(t) \propto \exp\left\{-\left[\frac{1/4\Delta\omega^2}{1/\Delta\omega^4+\varphi_2^2/4}\right]t^2\right\} = \exp\left\{-\left[\frac{1}{4/\Delta\omega^2+\Delta\omega^2\varphi_2^2}\right]t^2\right\}$$

So the pulse width is:

$$\Delta t = \sqrt{4/\Delta\omega^2+\Delta\omega^2\varphi_2^2}$$

I've neglected order-unity factors because we only care the phase required to double it.

What value of φ_2 is required to double the pulse length in the absence of chirp?

$$\begin{aligned}\sqrt{4/\Delta\omega^2+\Delta\omega^2\varphi_2^2} &= 2\sqrt{4/\Delta\omega^2}\\ 4/\Delta\omega^2+\Delta\omega^2\varphi_2^2 &= 16/\Delta\omega^2\\ \Delta\omega^2\varphi_2^2 &= 12/\Delta\omega^2\\ \varphi_2^2 &= 12/\Delta\omega^4\\ \varphi_2 &= 2\sqrt{3}/\Delta\omega^2\end{aligned}$$

But $\Delta\omega = 2\pi c\,\Delta\lambda/\lambda_0^2$

So the extra phase required to double the pulse due to chirp will be:

$$\varphi_2 = 2\sqrt{3}/\Delta\omega^2 = 2\sqrt{3}\,\lambda_0^4/\left(2\pi c\,\Delta\lambda\right)^2 = \sqrt{3}\,\lambda_0^4/2\pi^2c^2\Delta\lambda^2$$